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MODELING OF THE "FISH-HOOK" EFFECT IN A CLASSIFIER

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On the basis of the kinematic model of entrainment of small particles by large ones the abnormally nonmonotonic dependence of the separation curve on the particle size is explained. The influence of the particle density, the classifier length, the split-parameter, and the average size of particles on the depth of the "fish-hook" effect has been investigated.

Separation or classification of particles in technology is carried out on the basis of the process of natural sedimentation (precipitators) or under the action of the centrifugal force (hydrocyclones, centrifuges, etc.) [1–4]. To characterize the efficiency of separation of solid particles of a suspension in hydrocyclones, the separation curve which shows the portion of particles of each class of sizes (fractions) removed through the lower hole of the classifier is used. Ideally, the separation curve $T(d)$ is monotonically increasing from some value of $T(0) < 1$ at $d \to 0$ to $T = 1$ at $d \to \infty$. Such a shape of the curve follows from the classical notions about the dependence of the sedimentation rate of a particle on its size.

For many classifiers, such as hydrocyclones, $T(0) \neq 1$ and the separation curve with decreasing particle size is parallel to the x-axis, which is explained by the turbulent character of the flow [1, 3] in the classifier, as a result of which the small particles are uniformly distributed throughout the active volume and are removed in proportion to the suspension flow through the outlet. As the size of particles increases, their sedimentation rate increases and, therefore, they can be thrown onto the walls and removed from the apparatus. As opposed to such reasonings, many authors noted [5–9] that the separation curve for particles of size 10 μm and less has a minimum. Such behavior of the separation curve is called the "fish-hook" effect. Up to now this effect has not found a conclusive explanation. In the literature several variants of interpretation of the phenomenon are discussed:

1) insufficiently accurate granulometric analysis;
2) dependence of the density of materials on the particle size;
3) flocculation and agglomeration of a disperse material;
4) hydrodynamic interaction of particles of different sizes.

The first variant is associated with the relatively large spread of values in measuring sizes of the smallest particles with the use of the sedimentation or mesh analysis. The application of optical methods based on the laser beam scattering by particles permits avoiding this [5].

To explain variant 2), special experiments were performed with controlled homogeneous materials (carbon, aluminum, particles) [8] and the "fish-hook" effect was registered clearly.

Agglomeration and, accordingly, flocculation, as well as inverse processes, can undoubtedly take place in the course of hydrocycloning. To check this, special theoretical investigations and experiments were conducted [5, 6] with the use of surface-active substances and ultrasound to prevent agglomeration. The experiments have shown that precautions against agglomeration did not influence the characteristics of the "fish-hook" effect either; therefore, such a cause of the effect should be considered as unlikely.

The hydrodynamic mechanisms of the appearance of the "fish-hook" effect are widely discussed in the literature [5, 10–13], but mainly at the qualitative level, which only leads to the formulation of some empirical formulas. This helps to characterize the classification process, but does not contribute to the understanding of the phenomenon.

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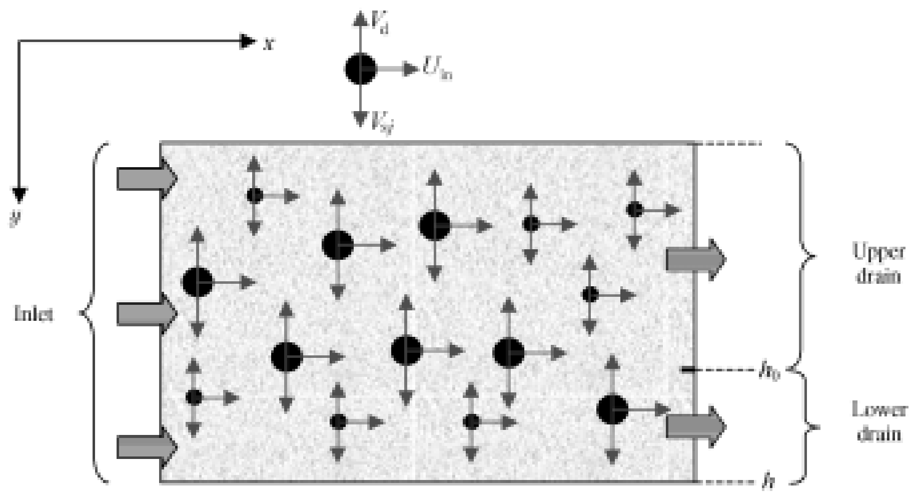


Fig. 1. Scheme of the classifier.

In the present work, with the help of the kinematic model of acceleration of sedimentation of small particles at the expense of large ones [14, 15] the abnormal behavior of the separation curve at small values of particle sizes is explained. In a simplified form, such an explanation was already given [16, 17] for apparatuses with a large residence time of particles in them. Below it will be shown that with decreasing length of the apparatus this effect weakens.

Model of the Classifier. Consider a classifier [18] of length l and height h, into which the suspension flows from the left at a constant velocity U_{in} and outflows from the right through the lower and upper drains (Fig. 1). The suspension particles are subjected to the action of a mass (gravitational or centrifugal) force along the y-axis and an opposite turbulent diffusion force. Thus, the particles in the classifier move from left to right and drift in the vertical direction.

Let V_{sj} be the velocity of sedimentation of a particle from the jth fraction, and $V_{dj} = -\frac{1}{c_j} D \frac{\partial c_j}{\partial y}$ be the transport velocity of particles as a result of turbulent diffusion. In this formulation, the turbulent diffusion coefficient of particles D is assumed to be a constant. Then the equation for determining the concentration of particles of each fraction d_j will be written in the form

$$U_{\text{in}} \frac{\partial c_j}{\partial x} + \frac{\partial}{\partial y}\left(V_{sj} c_j - D \frac{\partial c_j}{\partial y}\right) = 0 . \tag{1}$$

The initial conditions are: $x = 0$ at $c_j(0, y) = c_{j0}$. The boundary conditions are: at $y = 0$ and $y = h$ $V_{sj} c_j - D \frac{\partial c_j}{\partial y} = 0$.

The separation function characterizing the portion of particles of a given fraction removed through the lower drain, according to the separation model [1, 3], is defined as

$$T(d_j, x) = \frac{Q_{\text{un}j}(x)}{Q_{\text{un}j}(x) + Q_{\text{ov}j}(x)} . \tag{2}$$

Here $Q_{\text{un}j} = U_{\text{in}} \int_{h_0}^{h} c_j(x, y) dy$, $Q_{\text{ov}j} = U_{\text{in}} \int_{0}^{h_0} c_j(x, y) dy$ is the flow rate of the jth fraction of the solid material through the lower and upper holes of the apparatus.

In the absence of the solid phase the ratio between the liquid flow rate through the lower and upper holes of the apparatus (the so-called split-parameter) is $s = h_0/(h - h_0)$. The concentration fields and classification characteristics can be calculated if the sedimentation velocities of particles V_{sj} from the jth fraction are known.

Modeling of Particle Sedimentation in a Polydisperse Suspension. *Sedimentation velocity of a single particle*. In a stationary liquid, the sedimentation velocity of a single particle is defined by the Stokes formula

$$V_{\mathrm{St}j} = b\,\frac{g d_j^2}{18\mu}\,(\rho_\mathrm{p} - \rho_1)\,. \tag{3}$$

In the case of sedimentation of a single particle in a stationary suspension, in (3) instead of the liquid density and viscosity the suspension density $\rho_\mathrm{s} = \rho_1(1 - c_\mathrm{v}) + \rho_\mathrm{p} c_\mathrm{v}$ and viscosity $\mu_\mathrm{s} = (1 - c_\mathrm{v}/0.6)^{-1.5}$ are taken. Then the expression for the sedimentation velocity of the particle is written as follows:

$$V_{\mathrm{h}j} = V_{\mathrm{St}j}\,(1 - c_\mathrm{v})\,(1 - c_\mathrm{v}/0.6)^{1.5}\,. \tag{4}$$

Here $c_\mathrm{v} = \sum_j c_j$ is the total volume concentration of the solid fraction.

Entrainment of small particles by large ones. As small particles are moving in the vicinity of large ones, the effect of entrainment of small particles, which appears as an increase in the sedimentation velocity, can be observed. This effect was investigated theoretically on the basis of a cellular model in [14] for a rarefied bidisperse suspension, in which the particle size of one fraction is much larger than the size of particles of another fraction $d_i >> d_j$. In accordance with this model and in view of (4) the sedimentation velocity of small particles is defined by the formula

$$V_{\mathrm{e}j} = V_{\mathrm{h}j}\left(1 + A c_i^{1/3}\left(\frac{d_i}{d_j}\right)^2\right), \tag{5}$$

which for the case of a polydisperse suspension will take on the form

$$\left[d_j^2\left(\frac{V_{\mathrm{e}j}}{V_{\mathrm{h}j}} - 1\right)\right]^3 = A^3 c_i d_i^6\,. \tag{6}$$

The left-hand side of (6) is a function of parameters of the small fraction, and the right-hand one — of the large fraction. Assuming that each ith fraction, for which the condition $d_i >> d_j$ is fulfilled, makes a contribution to the processes of entrainment of the jth fraction in proportion to the complex $c_i d_i^6$, we rewrite formula (6) in the form

$$\left[d_j^2\left(\frac{V_{\mathrm{e}j}}{V_{\mathrm{h}j}} - 1\right)\right]^3 = A^3 \sum_{i>j} c_i d_i^6\,,$$

whence it follows that

$$V_{\mathrm{e}j} = V_{\mathrm{h}j}\left(1 + A c_\mathrm{v}^{1/3}\,\frac{1}{d_j^2}\left(\sum_{i>j} \Delta m_i d_i^6\right)^{1/3}\right). \tag{7}$$

Here $\sum_i \Delta m_i = 1$ and $c_i = c_\mathrm{v}\Delta m_i$.

The experiments conducted in [15, 19] have shown that the coefficient A in (7) depends on the volume fraction of the solid phase. In view of this, formula (7) will be written as

$$V_{ej} = V_{hj}\left(1 + g\,(c_v)\,\frac{f_e\,(d_j)}{d_j^2}\right), \qquad (8)$$

where $g(c_v) = 2.25\cdot c_v^{2/3}\,\exp\left[-\left(\frac{c_v}{0.2}\right)^2\right]$; $f_e(d_j) = \left(\sum_{i>j} \Delta m_i d_i^6\right)^{1/3}$ is the entrainment function.

The condition of selecting i for a given value of j can be obtained on the basis of the inequality derived for the bidisperse suspension in [14]: $d_i \geq \beta d_j$, where β is a constant, whose theoretical value confirmed by experiments in [19], is in the range from 10 to 15.

Liquid displacement by the sedimenting solid phase. In considering the sedimentation of a group of particles, whose volume fraction can no longer be ignored, the sedimentation velocty of a particle will differ from that obtained by (8) by the velocity value of the liquid being displaced V_{liq}:

$$V_{sj} = V_{ej} + V_{liq}\,. \qquad (9)$$

From the condition of total volume conservation follows

$$(1 - c_v)\,V_{liq} = -\sum_i c_i V_{si}\,. \qquad (10)$$

Eliminating the displaced liquid velocity from (9) and (10), we find that

$$V_{sj} = V_{ej} - \sum_i c_i V_{ei}\,. \qquad (11)$$

Total effect of the interaction of particles in the sedimentation process. Finally, the following expression for the sedimentation velocity of a particle in a polydisperse suspension from (8) and (11) is obtained:

$$V_{sj} = \frac{V_{hj}}{d_j^2}\left[d_j^2 + g\,(c_v)\,f_e\,(d_j) - c_v \sum_{i=1}^{n}\left(d_i^2 + g\,(c_v)\,f_e\,(d_i)\right)\Delta m_i\right]. \qquad (12)$$

If the size distribution of particles is given in the form of a continuous function $q(d)$ such that $\int_0^\infty q(s)ds = 1$, then the formula for the sedimentation rate of a particle (12) takes on the form

$$V_s\,(d) = \frac{V_h\,(d)}{d^2}\left[d^2 + g\,(c_v)\,f_e\,(\beta d) - c_v \int_0^\infty \left(s^2 + g\,(c_v)\,f_e\,(\beta s)\right) q\,(s)\,ds\right], \qquad (13)$$

where $f_e(d) = \left(\int_d^\infty q(s)s^6 ds\right)^{1/3}$.

The first terms in (12) and (13) correspond to the Stokes sedimentation velocity of a particle, the second ones correspond to the increase in the particle velocity due to its entrainment by larger particles, and the third terms are responsible for the influence on the particle sedimentation velocity of the flow of the liquid displaced by the sedimenting solid phase. As is seen from (12) and (13), the sedimentation velocity of a particle depends not only on its size, the medium properties, and the solid phase concentration in the suspension, but also on the size distribution function of particles.

In the present paper, we used the two-parameter RRSB (Rozen–Rammler–Sperling–Bennett) [1, 3] distribution function

$$q = \frac{n}{d_{\mathrm{m}}} \exp\left[-\left(\frac{d}{d_{\mathrm{m}}}\right)^{n}\right]\left(\frac{d}{d_{\mathrm{m}}}\right)^{n-1} . \tag{14}$$

Here d_{m} is the average size of particles in the suspension, and n characterizes the distribution sharpness.

Dimensionless Variables and Computing Method. Let us introduce the dimensionless variables

$$\theta_j = \frac{c_j}{c_{j0}} , \quad \xi = \frac{xD}{h^2 U_{\mathrm{in}}} , \quad \eta = \frac{y}{h} , \quad \varphi_j = \frac{d_j}{d_{\mathrm{m}}} , \quad W_j = \frac{V_{\mathrm{s}j}}{V_{\mathrm{St,m}}} ,$$

where $V_{\mathrm{St,m}}$ is the Stokes sedimentation velocity of a particle of size d_{m}. Then the system of equations (1) with initial and boundary conditions, as well as with closing equations for the sedimentation velocity (13), will be written as

$$\frac{\partial \theta_j}{\partial \xi} + \frac{\partial}{\partial \eta}\left(\mathrm{Pe_m} W_j \theta_j - \frac{\partial \theta}{\partial \eta}\right) = 0 \tag{15}$$

with the input conditions

$$\theta_j \Big|_{\xi=0} = 1 \tag{16}$$

and the boundary conditions

$$\text{at} \quad \eta = 0 \quad \text{and} \quad \eta = 1 \quad \mathrm{Pe_m} W_j \theta_j - \frac{\partial \theta_j}{\partial \eta} = 0 . \tag{17}$$

The equation for the dimensionless sedimentation velocity (14) will take on the form

$$W_j (\varphi_j, c_{\mathrm{v}}) = H (c_{\mathrm{v}}) \left\{\varphi_j^2 + g (c_{\mathrm{v}}) f\left((\beta\varphi_j)^m\right) - c_{\mathrm{v}} \int_0^{\infty} \left(t^{2/m} + g (c_{\mathrm{v}}) f (\beta^m t)\right) \exp(-t)\, dt\right\} ,$$

where

$$f(x) = \left(\int_x^{\infty} t^{6/m} \exp(-t)\, dt\right)^{1/3} ; \quad H (c_{\mathrm{v}}) = (1 - c_{\mathrm{v}}) \left(1 - \frac{c_{\mathrm{v}}}{0.6}\right)^{1.5} ; \quad c_{\mathrm{v}} (\xi, \eta) = \sum_{j=1}^{n} c_{j0} \theta_j (\xi, \eta) .$$

$\mathrm{Pe_m} = \dfrac{hV_{\mathrm{St,m}}}{D}$, $c_{\mathrm{v}0} = \sum\limits_{j=1}^{n} c_{j0}$, s denotes the diagnostic variables of the problem. The dimensionless length of the apparatus is given by the parameter $L = \dfrac{lD}{h^2 U_{\mathrm{in}}}$.

Integrating (15) with respect to η from 0 to 1 and using the input conditions (16) and the boundary conditions (17), we obtain

$$\int_0^1 \theta_j (\xi, \eta)\, d\eta = 1 . \tag{18}$$

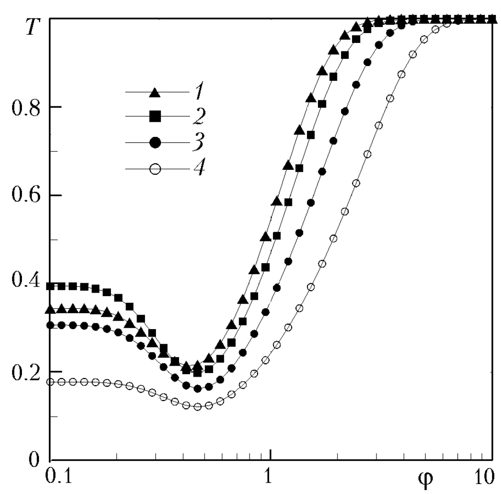


Fig. 2. Influence of the solid phase concentration on the separation function in the outlet cross-section L = 1: 1) c_v = 0.02; 2) 0.04; 3) 0.08; 4) 0.16.

Then the separation function (2) in dimensionless form will take on the form

$$T(d_j, \xi) = \int_{\frac{s}{s+1}}^{1} \theta_j(\xi, \eta)\, d\eta \,. \tag{19}$$

If the concentration of the solid phase of the suspension fed into the apparatus is very small, then there will be no effects of interaction of particles, and the equations for each fraction can be solved independent of one another. For comparison, let us give the solution for a low-concentration suspension in the case of an infinitely long apparatus:

$$\theta_j(\infty, \eta) = \frac{\mathrm{Pe}_j}{\exp(\mathrm{Pe}_j) - 1} \exp(\mathrm{Pe}_j\, \eta) \,, \tag{20}$$

on whose basis function (19) will be defined as $T(\mathrm{Pe}_j) = \dfrac{1 - \exp\left(-\dfrac{\mathrm{Pe}_j}{1+s}\right)}{1 - \exp(-\mathrm{Pe}_j)}$. The latter is a function monotonically increasing from $T(0) = \dfrac{1}{1+s}$ at $\mathrm{Pe_m}\, W_j \to 0$ to 1 at $\mathrm{Pe_m}\, W_j \to \infty$.

The calculations were performed at the following values of the parameters: β = 10, m = 1.7. The total number of fractions N was taken to be equal to 40, and the dimensionless diameter of particles of the jth fraction was determined by the rule $\varphi_j = 0.1 \cdot 100^{\frac{j-1}{N-1}}$.

For the numerical solution of Eq. (15), we used the Patankar difference scheme [20] with an exponential representation of the solution for θ_j at the cell boundaries in the approximation of the convective part of the equation. Since the sedimentation velocity of particles depends on the concentration, then to find $W_j(\varphi_j, c_v)$, at each step of integration of Eq. (16) with respect to ξ internal iterations were organized. The value of the computational mesh width in the longitudinal direction was assumed to be equal to 0.01, and in the transverse direction — to 0.025.

In the course of calculations, the following parameters were varied: the output cross-section ratio (parameter s), the concentration of the solid phase of the suspension fed into the apparatus c_v, and the average size of particles in the suspension (parameter $\mathrm{Pe_m}$).

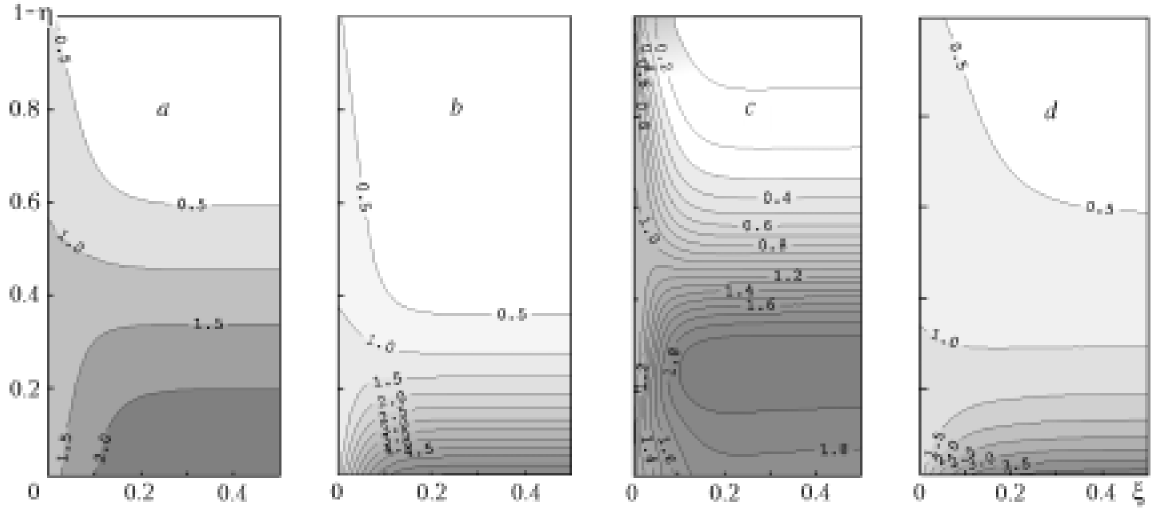


Fig. 3. Fields of the relative concentration θ_j of particles of diameter φ = 0.94 (a, b) and φ = 0.1 (c, d) in the classifier at various values of the initial concentration of the solid phase: a and b) c_v = 0.16; c and d), 0.02. Pe_m = 10.

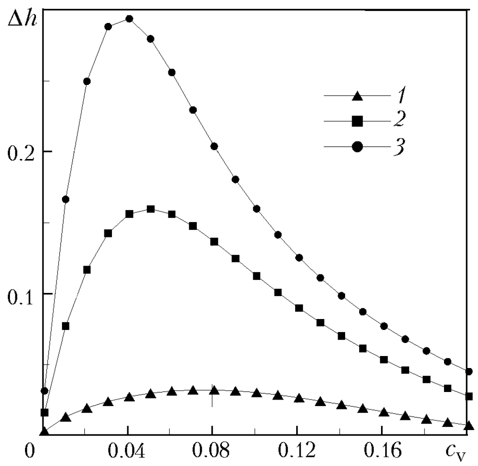
Results of the Calculations and Discussion. Figure 2 shows the behavior of the separation curves calculated at the output from the classifier (L = 1) at various values of the initial concentration of the solid phase of the suspension obtained for Pe_m = 10 and s = 9.

It is seen that for particles of size larger than $\varphi \approx 0.5$ the separation function monotonically increases. The share of particles with sizes $0 < \varphi$ 5 removed through the lower drain monotonically decreases with increasing φ, which is explained by the carrying out of small particles as they are entrained by larger ones. An increase in the initial concentration of the solid phase in the suspension leads to a monotonic decrease in the values of the separation function in the region of $0.5 < \varphi < 10$, which is due to the decrease in the sedimentation velocity of particles because of the hemmed conditions of sedimentation that decreases the relative concentration of large particles in the lower outlet (Fig. 3a, b). In so doing, the separation grain (the value of φ at which $T(\varphi)$ = 0.5) shifts towards larger particles — a well-known fact from experiments and practice [1–3].

For particles of size smaller than 0.5 the dependence of the separation function on the total concentration of the solid phase of the fed suspension becomes nonmonotonic. For a diluted suspension, with increasing concentration of particles the number of entrained small particles carried out together with large ones through the lower drain (Fig. 3a) increases at an almost invariable sedimentation velocity of the latter, which leads to an increase in the separation function. At high values of the inlet concentration of particles the sedimentation velocity of particles decreases because of the hemmed sedimentation conditions, which leads to a decrease in the relative share of the large particles carried out through the lower drain (Fig. 3a) and, consequently, of the small particles entrained by them (Fig. 3c).

To characterize the observed phenomenon, let us introduce the notion of the depth of the "fish-hook" effect as the difference $\Delta h = T(0.1) - \frac{1}{1+s}$. Figure 4 shows the dependence of Δh on the initial concentration of the solid phase at various values of the Peclet number Pe_m, which is nonmonotonic with a clearly defined maximum. Such behavior of the effect depth is explained by two opposite factors: entrainment of small particles by large ones, on the one hand, and retardation of the sedimentation process due to the increase in the suspension viscosity and density, on the other. From Fig. 4 it is seen that an increase in Pe_m, which corresponds to a decrease in the diffusion coefficient of particles or an increase in their intrinsic sedimentation velocity, leads to an increase in the maximum of Δh and its displacement into the region of low values of the initial concentration. Here the influence of the particle size distribution function on the separation characteristics is apparent.

The dependence of Δh on the initial concentration of the solid phase at various values of the Peclet number Pe_m that arises in the calculations completely agrees qualitatively with the results presented in [5]. For the two apparatuses with working cylinder diameters of 25 and 39 mm used in the experiments, a maximum of Δh at c_v = 0.04



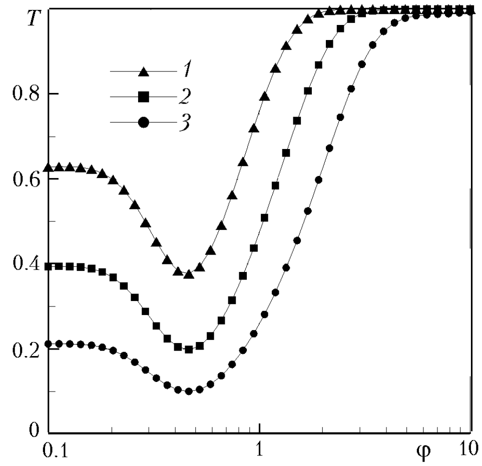
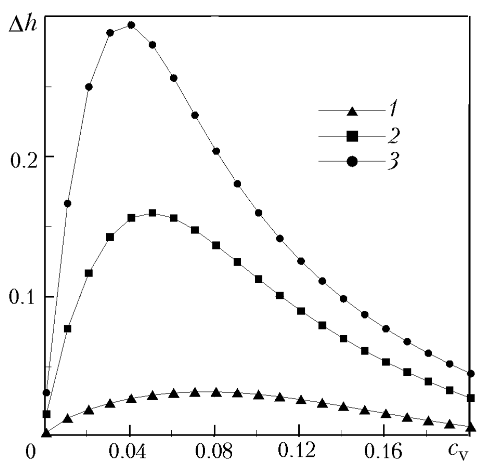


Fig. 4. Dependence of the "fish-hook" effect depths in the outlet cross-section $L = 1$ on the initial concentration of particles: 1) $\mathrm{Pe_m}$; 2) 5; 3) 10. $s = 0.9$.

Fig. 5. Influence of the split-parameter on the separation function: 1) $s = 4$; 2) 9; 3) 19. $\mathrm{Pe_m} = 10$; $c_v = 0.04$, $L = 1$.



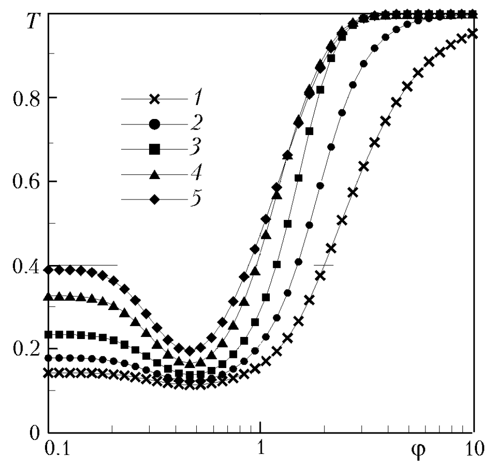


Fig. 6. Dependence of the "fish-hook" effect depth on the initial concentration of particles: 1) $s = 4$; 2) 9; 3) 19. $\mathrm{Pe_m} = 10$, $L = 1$.

Fig. 7. Influence of the classifier length on the separation function: 1) $L =$ 0.01; 2) 0.02; 3) 0.04; 4) 0.1; 5) 0.3; $s = 9$, $c_v = 0.04$, $\mathrm{Pe_m} = 10$.

has been observed steadily, and its value decreased by a factor of 1.9 upon replacement of the larger hydrocyclone by a smaller one, i.e., upon a change (in our notation) in $\mathrm{Pe_m}$ by a factor of 1.6. While in Fig. 4 a much stronger dependence on $\mathrm{Pe_m}$ follows, it should be taken into account that a change in the hydrocyclone size, with the pressure constant, leads to a drastic change in the flow field, which is not reflected in the calculations.

An increase in the split-parameter caused by a decrease in the size of the lower drain leads to not only a decrease in the share of small and medium particles carried out through the lower drain (Fig. 5), but also to a decrease in the depth of the "fish-hook" effect (Fig. 6). This can be explained by the fact that such a change in the size of the lower drain leads to a decrease in the number of fractions entraining small particles. The largest particles are concentrated at the bottom of the apparatuses. At a very large s, i.e., at a narrow lower hole only the largest particles participate in entraining small ones. The influence of s described above is also in good agreement with the experimental measurement data presented in [5]. From Fig. 3 it is seen that the concentration fields are formed at a distance from the inlet $\xi = 0.1$. Therefore, the data presented in Figs. 2–6 are characteristic of the given apparatus.

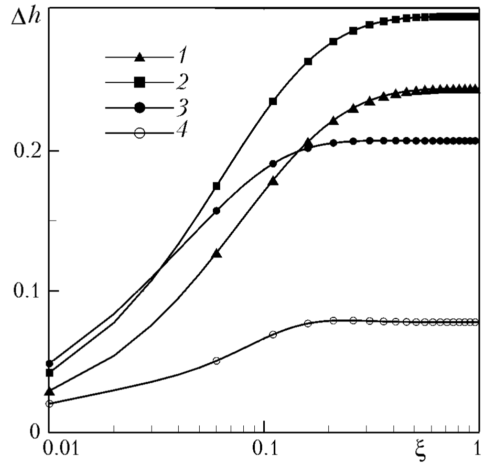


Fig. 8. Dependence of the "fish-hook" effect depth on the classifier length at various volume concentrations of the solid material: 1) c_v = 0.02; 2) 0.04; 3) 0.08; 4) 0.16. s = 9, Pe_m = 10.

The influence of the classifier length on the character of the separation curve behavior and the "fish-hook" effect depth is shown in Figs. 7 and 8, respectively. With increasing length of the apparatus the portion of particles removed through the lower drain increases to L = 0.3, and then remains unaltered (Fig. 7). From Fig. 8 it is seen that with increasing inlet concentration of the solid phase the depth of the "fish-hook" effect reaches a constant value at smaller values of L. Such behavior of the $T(\varphi)$ and $\Delta h(\xi)$ curves is explained by the fact that for short apparatuses equilibrium between the sedimentation and diffusion flows has no time to be established.

CONCLUSIONS

1. The formula for increase in the sedimentation velocity of small particles due to their entrainment by large ones has been generalized to the case of a polydisperse suspension.

2. With the aid of the kinematic model of entrainment of small particles by large ones the abnormal behavior of the separation curve at small values of particle sizes, the so-called "fish-hook" effect has been explained.

3. It has been established that the depth of the "fish-hook" effect of the separation curve as a function of the inlet concentration of particles is nonmonotonic: it increase with increasing parameter Pe_m and classifier length L and with decreasing split-parameter s. This conclusion agrees with the experiments described in [5].

4. It has been shown that the portion of small particles with sizes $d \leq 0.5 d_m$ carried out through the lower drain increases with increasing classifier length and the average size of particles in the suspension, as well as with decreasing value of the split-parameter. The dependence of this portion on the inlet concentration of the solid phase is nonmonotonic.

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NOTATION

A, coefficient; b, centrifugal number (ratio of the centrifugal acceleration in the hydrocyclone to the gravitational one); d, diameter of particles, m; D, turbulent diffusion coefficient, m^2/sec; c, particle concentration; f_e, entrainment function, m^2; g, gravitational acceleration, m/sec^2; h, height of the classifier, m; h_0, height of the upper drain, m; l, length of the classifier, m; Δm, relative volume concentration of particles; Pe, Peclet number; q, particle size distribution function, m^{-1}; Q, flow velocity, m^2/sec; s, split-parameter; T, separation function; U, V, velocity, m/sec; W, dimensionless sedimentation rate; x and y, longitudinal and transverse coordinates, m; μ, dynamic viscosity, Pa·sec; ρ, density, kg/m^3; θ, dimensionless concentration; ξ and η, dimensionless longitudinal and transverse coordinates; φ, dimensionless particle diameter. Subscripts: 0, initial condition; d, diffusion; e, entrainment; h, hemming; *i*, *j*, fraction numbers; in, inlet; liq, liquid; ov, upper drain; p, particle material; s, suspension; un, lower drain; m, mean value; v, volume.

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